

Kinetic equations for a nonideal quantum system

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In the framework of real-time Green's functions, the general kinetic equations are investigated in a first-order gradient expansion. Within this approximation, the problem of the reconstruction of the two-time correlation functions from the one-time Wigner function was solved. For the Wigner function, a cluster expansion is found in terms of a quasiparticle distribution function. In equilibrium, this expansion leads to the well-known generalized Beth-Uhlenbeck expression of the second virial coefficient. As a special case, the T -matrix approximation for the self-energy is investigated. The quantum kinetic equation derived thus has, besides the (Markovian) Boltzmann collision integral, additional terms due to the retardation expansion which reflect memory effects. Special interest is paid to the case that bound states exist in the system. It is shown that the bound state contribution, which can be introduced via a bilinear expansion of the two-particle T matrix, follows from the first-order retardation term in the general kinetic equation. The full Wigner function is now a sum of one function describing the unbound particles and another one for the bound state contribution. The latter two functions have to be determined from a coupled set of kinetic equations. In contrast to the quantum Boltzmann equation, energy and density of a nonideal system are conserved. [S1063-651X(96)11009-6]

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I. INTRODUCTION

The kinetic equations which are mostly used in the analysis of nonequilibrium properties of plasmas, semiconductors, and nuclear matter are the famous Boltzmann equation or other Boltzmann-like kinetic equations such as the Landau equation and the Lenard-Balescu equation. Though the Boltzmann equation is one of the fundamental equations in statistical physics, as it describes the irreversible relaxation to stationary states and because it is the basis of the transport theory, this equation has many principal shortcomings. Two of these are (i) the Boltzmann equation is valid on time scales larger than the correlation time τ^{corr} only, and (ii) the Boltzmann equation conserves the kinetic energy or the quasiparticle energy only, instead of the total (kinetic plus potential) energy.

The second point especially is a serious drawback in strongly correlated many-particle systems. In such systems, thermodynamic functions, as, for example, the internal energy, are essentially determined by correlation effects. The Boltzmann equation describes the relaxation towards the stationary states under the condition of the conservation of the mean kinetic energy. This is unphysical. Any kinetic theory of strongly correlated systems has to describe this relaxation to the nonideal thermodynamic properties. Now it is well known since papers by Bärwinkel [1,2] and Klimontovich [3] that this defect of Boltzmann-type equations is essentially connected with the approximations with respect to the time. Boltzmann-like kinetic equations are approximations local in time (Markovian equations) when derived from the most general nonlocal form of kinetic equations given by Prigogine [4], Resibois [5], Zwanzig [6], Kadanoff and Baym [7] and others. In the latter papers, attempts were done to overcome the shortcomings, and generalizations of Boltzmann-

like Markovian equations were given. For further references see, e.g., [8–13].

A very powerful method for this purpose is the real-time Green's function technique, which allows to describe the properties of strongly correlated system in equilibrium as well as in nonequilibrium in a consistent manner. Within the framework of real-time Green's functions, the equilibrium and nonequilibrium properties of a many-particle system are determined by the two-time correlation functions $\pm ig^{\langle}(1,1') = \langle \Psi^+(1') \Psi(1) \rangle$ and $ig^{\rangle}(1,1') = \langle \Psi(1) \Psi^+(1') \rangle$ with the abbreviation $1 = r_1, s_1^3, t_1$. These functions contain the statistical and spectral information on the system. For $t_1 = t_1'$ the correlation function is just the single-particle density matrix. The difference of g^{\rangle} and g^{\langle} gives the spectral function.

In thermodynamic equilibrium, it is sufficient to determine the spectral properties only because the statistical operator is known. In order to take into account correlation contributions, an approximation beyond the quasiparticle approximation is necessary. In [14], an expansion with respect to the damping was performed and a generalized Beth-Uhlenbeck [15] expression for the density was obtained. This is possible for the total energy, too. The aim of this paper is to derive kinetic equations on the same level of approximation. That means that the conserved quantities describe nonideal quantum systems up to the order of a generalized second virial coefficient.

The known problem to get kinetic equations from Green's functions is the fact that the equation for the (single-time) density matrix or the Wigner function, which follows from the dynamic equation for g^{\langle} , is not closed. The reason is that both of the two-time correlation functions g^{\langle} and g^{\rangle} arise on the right hand side. Further these equations are nonlocal in space and time. In this paper, we will consider the equa-

tions in first-order gradient expansion.

In order to get a kinetic equation in explicit form, one has to solve still two problems: (i) The self-energy Σ has to be determined as a functional of g^{\cong} by perturbation theory. Here the standard two-particle collision approximation (T -matrix approximation) will be used. (ii) The two-time correlation functions have to be reconstructed from the single-time single-particle density matrix. This is the so-called reconstruction problem, which was addressed first by Lipavský, Špička, and Velický [16].

The general solution for the reconstruction of the g^{\cong} functions by the single-particle density matrix was given in [16] by a complicated integral equation. Usually only the generalized Kadanoff-Baym (GKB) ansatz $g^{<} = g^R f^{<} + f^{<} g^A$ is used. This ansatz is valid only in quasiparticle approximation, but here a kinetic theory will be considered which goes beyond this approximation.

The plan of this paper is outlined as follows. Starting from the basic equations, the nonequilibrium spectral function is considered in an extended quasiparticle approximation in Sec. II. With the solution of the reconstruction problem (Sec. III), we derive, from the Kadanoff-Baym equations, the first-order gradient form of the kinetic equation for the Wigner distribution. In Sec. IV, we use the binary collision approximation for Σ^{\cong} , the first-order gradient expansion for the optical theorem and the solution of the reconstruction problem in first-order gradient expansion. In this way we obtain nonideality corrections to the Boltzmann equation. We will show for a nondegenerate quantum system that the kinetic equations derived give correct conservation laws for the number density and the energy density in the binary collision approximation. In Sec. V, systems will be considered in which bound states are possible. It will be shown that bound state contributions occur in the first-order retardation term in the general kinetic equation.

II. EXTENDED QUASIPARTICLE APPROXIMATION FOR THE NONEQUILIBRIUM SPECTRAL FUNCTION

Within the framework of real-time Green's functions, the nonequilibrium properties of a many-particle system are determined by the two-time correlation functions

$$\begin{aligned} \pm i g^{<}(1,1') &= \langle \Psi^+(1') \Psi(1) \rangle, \\ i g^{>}(1,1') &= \langle \Psi(1) \Psi^+(1') \rangle \end{aligned} \quad (2.1)$$

with the abbreviation $1 = \mathbf{r}_1, t_1$ (the spin is suppressed in the following). Ψ^+ and Ψ are creation and annihilation operators, respectively. The brackets mean the averaging with the density operator of the system. The single-particle Wigner distribution function is given by

$$f(\mathbf{p}, \mathbf{R}, T) = \int \frac{d\omega}{2\pi} (\pm i) g^{<}(\mathbf{p}, \omega, \mathbf{R}, T) \quad (2.2)$$

with $g^{<}(\mathbf{p}, \omega, \mathbf{R}, T)$ being the Fourier transform of $g^{<}(1,1') = g^{<}(\mathbf{r}, t, \mathbf{R}, T)$ with respect to the variables $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}'_1$ and $t = t_1 - t'_1$, respectively. \mathbf{R} and T are determined by $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}'_1)/2$ and $T = (t_1 + t'_1)/2$.

The dynamic equations for the nonequilibrium correlation functions $g^{<}$ and $g^{>}$ were derived first by Kadanoff and

Baym [7] and by Keldysh [17]. A derivation, starting from the Martin-Schwinger-Hierarchy of equations of motion for the n -particle Green's functions [18,19] and using the condition of weakening of initial correlations was given in [20]. In a convenient way, this system of equations is written in the following form [21,22]:

$$\begin{aligned} \left(i \frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} \right) g^{\cong}(1,1') - \int_{-\infty}^{\infty} d\bar{1} \Sigma^R(1,\bar{1}) g^{\cong}(\bar{1},1') \\ = \int_{-\infty}^{\infty} d\bar{1} \Sigma^{\cong}(1,\bar{1}) g^A(\bar{1},1'), \\ \left(i \frac{\partial}{\partial t_1} + \frac{\nabla_1^2}{2m} \right) g^{R/A}(1,1') - \int_{-\infty}^{\infty} d\bar{1} \Sigma^{R/A}(1,\bar{1}) g^{R/A}(\bar{1},1') \\ = \delta(1-1'), \end{aligned} \quad (2.3)$$

where Σ is the self-energy function.

The equation for the Wigner function follows from (2.3) and its adjoint equation in the case $t_1 = t'_1 = T$, and for a spatially homogeneous system it reads

$$\begin{aligned} \frac{\partial f(\mathbf{p}, T)}{\partial T} = \int_{-\infty}^T d\bar{t} [\Sigma^{<}(T, \bar{t}) g^A(\bar{t}, T) + \Sigma^R(T, \bar{t}) g^{<}(\bar{t}, T) \\ - g^{<}(T, \bar{t}) \Sigma^A(\bar{t}, T) - g^R(T, \bar{t}) \Sigma^{<}(\bar{t}, T)]. \end{aligned} \quad (2.4)$$

This kinetic equation for the Wigner function is not closed because the single-particle Green's function arises on the right hand side. To get a closed equation one would like to have an expression of the Green's function in terms of the Wigner function. This is also called the reconstruction problem and was addressed by Lipavský *et al.* in [16]. We will come back to this point in Sec. III.

The Kadanoff-Baym equations are nonlocal equations in space and time. Applying the so-called gradient expansion [7,22] to (2.3) and the adjoint equation, one obtains

$$\{ \text{Reg}^{R-1}, i g^{<(>)} \} - \{ i \Sigma^{<(>)}, \text{Reg}^R \} = g^{<} \Sigma^{>} - g^{>} \Sigma^{<}, \quad (2.5)$$

where the quantities are functions of $\mathbf{p}, \omega, \mathbf{R}, T$, and Poisson brackets were defined by

$$\{A, B\} = \frac{\partial A}{\partial \omega} \frac{\partial B}{\partial T} - \frac{\partial A}{\partial T} \frac{\partial B}{\partial \omega} - \frac{\partial A}{\partial \mathbf{p}} \frac{\partial B}{\partial \mathbf{R}} + \frac{\partial A}{\partial \mathbf{R}} \frac{\partial B}{\partial \mathbf{p}}. \quad (2.6)$$

The equation for the retarded (advanced) Green's functions on the same level of approximation reads

$$\left(\omega \pm i\varepsilon - \frac{p^2}{2m} - \Sigma^{R/A}(\mathbf{p}, \omega, \mathbf{R}, T) \right) g^{R/A}(\mathbf{p}, \omega, \mathbf{R}, T) = 1. \quad (2.7)$$

Here the first derivatives cancel each other.

At this point, one has to note that the set of Eqs. (2.5) and (2.7) determines the statistical and the spectral properties of the particles as well. It turns out that the spectral properties are described by the spectral function, defined by

$$a(\mathbf{p}, \omega, \mathbf{R}, T) = i(g^{>} - g^{<}) = i(g^R - g^A). \quad (2.8)$$

In the gradient approximation given by (2.5) and (2.7), we get

$$a(\mathbf{p}, \omega, \mathbf{R}, T) = \frac{\Gamma(\mathbf{p}, \omega, \mathbf{R}, T) + 2\varepsilon}{[\text{Reg}^{R^{-1}}(\mathbf{p}, \omega, \mathbf{R}, T)]^2 + [\frac{1}{2}\Gamma(\mathbf{p}, \omega, \mathbf{R}, T) + \varepsilon]^2}, \quad (2.9)$$

where the damping $\Gamma = i(\Sigma^> - \Sigma^<) = -2 \text{Im}\Sigma^R$ was introduced. Expression (2.9) has the same shape as the equilibrium one, but it is valid locally at \mathbf{R}, T . The main contribution to the spectral function comes from frequencies where we have $\text{Reg}^{R^{-1}} = 0$, i.e., from solutions of the quasiparticle dispersion relation

$$E(\mathbf{p}, \mathbf{R}, T) = \frac{p^2}{2m} + \text{Re}\Sigma(\mathbf{p}, \omega, \mathbf{R}, T)|_{\omega=E(\mathbf{p}, \mathbf{R}, T)}. \quad (2.10)$$

Following Kremp *et al.* [14], one can, in the case that the quasiparticle concept may be applied, expand the spectral function with respect to the small damping Γ , with the result

$$a(\mathbf{p}, \omega, \mathbf{R}, T) = 2\pi\delta\left(\omega - \frac{p^2}{2m} - \text{Re}\Sigma(\mathbf{p}, \omega, \mathbf{R}, T)\right) - \left(\frac{P'}{\omega - \frac{p^2}{2m} - \text{Re}\Sigma(\mathbf{p}, \omega, \mathbf{R}, T)}\right) \times \Gamma(\mathbf{p}, \omega, \mathbf{R}, T), \quad (2.11)$$

with $P'(1/x) = \lim_{\varepsilon \rightarrow 0} (\partial/\partial x)[x/(x^2 + \varepsilon^2)]$ being the derivative of the principal value.

The real part of the retarded self-energy is connected with the imaginary part via the dispersion relation

$$\text{Re}\Sigma(\mathbf{p}, \omega, \mathbf{R}, T) = \Sigma^{\text{HF}}(\mathbf{p}, \mathbf{R}, T) + P \int \frac{d\bar{\omega}}{2\pi} \frac{\Gamma(\mathbf{p}, \bar{\omega}, \mathbf{R}, T)}{\omega - \bar{\omega}}. \quad (2.12)$$

Thus, to be consistent, the quasiparticle renormalization factor is expanded, and we get

$$a(\mathbf{p}, \omega, \mathbf{R}, T) = 2\pi\delta(\omega - E(\mathbf{p}, \mathbf{R}, T)) \times \left(1 + \frac{\partial}{\partial \omega} \text{Re}\Sigma(\mathbf{p}, \omega, \mathbf{R}, T)|_{\omega=E}\right) - \left(\frac{\partial}{\partial \omega} P \frac{1}{\omega - E}\right) \Gamma(\mathbf{p}, \omega, \mathbf{R}, T). \quad (2.13)$$

This approximation for the spectral function fulfills the sum rule.

The question arises which approximations have to be done in the kinetic equation (2.5) to be consistent with the approximation (2.11). This will be essential in what follows in the next chapters. Subtracting the kinetic equations (2.5) for $g^>$ and $g^<$ from each other, one gets

$$\{\text{Reg}^{R^{-1}}, a\} - \{\Gamma, \text{Reg}^R\} = 0. \quad (2.14)$$

One can show that this equation remains true for the spectral function in approximation (2.11) if the real part of the retarded Green's function is considered in the limit $\Gamma = 0$:

$$\text{Reg}^R(\mathbf{p}, \omega, \mathbf{R}, T) = P \frac{1}{\omega - \frac{p^2}{2m} - \text{Re}\Sigma^R(\mathbf{p}, \omega, \mathbf{R}, T)}. \quad (2.15)$$

Thus we will use in the following this approximation for Reg^R .

III. RECONSTRUCTION OF THE CORRELATION FUNCTIONS

In this section, the connection between the correlation functions $g^<$ and $g^>$ and the density matrix in Wigner representation shall be investigated. Starting point is the generalized kinetic equation (2.5). For the sake of clarity, we drop for a moment the derivatives with respect to \mathbf{p} and \mathbf{R} . The derivatives in Eq. (2.5) can be rearranged to give

$$\frac{\partial}{\partial T} \left[ig^{\cong} - \frac{\partial \text{Re}\Sigma^R}{\partial \omega} ig^{\cong} + i\Sigma^{\cong} \frac{\partial \text{Reg}^R}{\partial \omega} \right] + \frac{\partial}{\partial \omega} \left[\frac{\partial \text{Re}\Sigma^R}{\partial T} ig^{\cong} - i\Sigma^{\cong} \frac{\partial \text{Reg}^R}{\partial T} \right] = g^<\Sigma^> - g^>\Sigma^<. \quad (3.1)$$

Taking approximation (2.15) for Reg^R , one gets

$$\frac{\partial}{\partial T} \left[\left(1 - \frac{\partial \text{Re}\Sigma^R}{\partial \omega}\right) \left(ig^{\cong} + i\Sigma^{\cong} P' \frac{1}{\omega - \frac{p^2}{2m} - \text{Re}\Sigma^R} \right) \right] + \frac{\partial}{\partial \omega} \left[\frac{\partial \text{Re}\Sigma^R}{\partial T} \left(ig^{\cong} + i\Sigma^{\cong} P' \frac{1}{\omega - \frac{p^2}{2m} - \text{Re}\Sigma^R} \right) \right] = g^<\Sigma^> - g^>\Sigma^<. \quad (3.2)$$

The structure of this equation now suggests to define a new quantity Q^{\cong} by

$$iQ^{\cong} = ig^{\cong} + i\Sigma^{\cong} P' \frac{1}{\omega - \frac{p^2}{2m} - \text{Re}\Sigma^R}, \quad (3.3)$$

which leads to

$$\frac{\partial}{\partial T} \left[\left(1 - \frac{\partial \text{Re}\Sigma}{\partial \omega}\right) iQ^{\cong} \right] + \frac{\partial}{\partial \omega} \left[\frac{\partial \text{Re}\Sigma}{\partial T} iQ^{\cong} \right] = Q^<\Sigma^> - Q^>\Sigma^<. \quad (3.4)$$

In order to find the physical meaning of the special correlation functions Q^{\cong} we introduce the spectral function according to the usual definition by

$$A_Q = i(Q^> - Q^<) = 2\pi\delta\left(\omega - \frac{p^2}{2m} - \text{Re}\Sigma^R\right). \quad (3.5)$$

To get the last line, Eqs. (2.8) and (2.11) were used. From the representation (3.5), it turns out that the quantity Q de-

scribes quasiparticles [23,24]. It is useful to introduce a quasiparticle distribution function by

$$\begin{aligned} \pm iQ^<(\mathbf{p}, \omega, \mathbf{R}, T) &= 2\pi\delta\left(\omega - \frac{p^2}{2m} - \text{Re}\Sigma^R\right) f^Q(\mathbf{p}, \mathbf{R}, T) \\ &+ X(\mathbf{p}, \omega, \mathbf{R}, T). \end{aligned} \quad (3.6)$$

For $Q^>$ we get

$$\begin{aligned} iQ^>(\mathbf{p}, \omega, \mathbf{R}, T) &= 2\pi\delta\left(\omega - \frac{p^2}{2m} - \text{Re}\Sigma^R\right) [1 \pm f^Q(\mathbf{p}, \mathbf{R}, T)] \\ &+ X(\mathbf{p}, \omega, \mathbf{R}, T) \end{aligned} \quad (3.7)$$

with a function $X(\mathbf{p}, \omega, \mathbf{R}, T)$ arbitrary so far.

The determination of the quantity X is the point where the problem of an ansatz arises. As stated above, the correlation functions Q^{\cong} describe undamped quasiparticles. If these functions are considered in the time domain, an ansatz similar to the generalized Kadanoff-Baym ansatz can be used:

$$\pm Q^<(t, t') = G_Q^R(t, t') f^Q(t') - f^Q(t) G_Q^A(t, t') \quad (3.8)$$

with f^Q and $G_Q^{R/A}$ being the quasiparticle distribution function and the quasiparticle propagators, respectively. Introducing the difference time τ and the centered time T in the usual way and performing an expansion with respect to the retardation in the quasiparticle distribution, we get after Fourier transformation

$$\pm iQ^<(\omega, T) = A_Q(\omega, T) f^Q(T) - \frac{\partial}{\partial\omega} \text{Re}G_Q^R(\omega, T) \frac{\partial}{\partial T} f^Q(T). \quad (3.9)$$

The correlation function $Q^>$ is given in a similar way by

$$\begin{aligned} iQ^>(\omega, T) &= A_Q(\omega, T) [1 \pm f^Q(T)] - \frac{\partial}{\partial\omega} \text{Re}G_Q^R(\omega, T) \\ &\times \frac{\partial}{\partial T} [1 \pm f^Q(T)]. \end{aligned} \quad (3.10)$$

Comparison with (3.6), (3.7) gives

$$X(\omega, T) = -\frac{\partial}{\partial\omega} \text{Re}G_Q^R(\omega, T) \frac{\partial}{\partial T} [\pm f^Q(T)]. \quad (3.11)$$

If we insert (3.9) in (3.3), the following expression in terms of the quasiparticle distribution function can be derived for $g^<$

$$\begin{aligned} \pm ig^<(\omega, T) &= 2\pi\delta\left(\omega - \frac{p^2}{2m} - \text{Re}\Sigma^R\right) f^Q(T) \\ &- P' \frac{1}{\left(\omega - \frac{p^2}{2m} - \text{Re}\Sigma\right)} (\pm i)\Sigma^<(\omega, T) \\ &- P' \frac{1}{\omega - \frac{p^2}{2m} - \text{Re}\Sigma(\omega)} \frac{\partial}{\partial T} [\pm f^Q(T)]. \end{aligned} \quad (3.12)$$

A similar expression follows for $g^>$. As can be seen from this expression, the correlation functions consist of a pole contribution and an off-pole part.

Now it is easy to obtain the full Wigner function. Integration of (3.12) with respect to ω and expansion of the renormalization factor gives

$$\begin{aligned} f(\mathbf{p}, \mathbf{R}, T) &= f^Q(\mathbf{p}, \mathbf{R}, T) \left(1 + \frac{\partial}{\partial\omega} \text{Re}\Sigma(\mathbf{p}, \omega, \mathbf{R}, T)|_{\omega=E}\right) \\ &- \int \frac{d\omega}{2\pi} P' \frac{1}{\left(\omega - \frac{p^2}{2m} - \text{Re}\Sigma\right)} (\pm i)\Sigma^<(\omega, T). \end{aligned} \quad (3.13)$$

If the dispersion relation (2.12) is applied, we get

$$\begin{aligned} f(\mathbf{p}, \mathbf{R}, T) &= f^Q(\mathbf{p}, \mathbf{R}, T) \\ &+ \int \frac{d\omega}{2\pi} P' \frac{1}{(\omega - E)} \{i\Sigma^>(\mathbf{p}, \omega, \mathbf{R}, T) f^Q(\mathbf{p}, \mathbf{R}, T) \\ &- (\pm i)\Sigma^<(\mathbf{p}, \omega, \mathbf{R}, T) [1 \pm f^Q(\mathbf{p}, \mathbf{R}, T)]\}. \end{aligned} \quad (3.14)$$

With Eqs. (3.13) and (3.14), interesting relations were obtained which make it possible to determine the Wigner function f from the quasiparticle distribution function f^Q . The first term on the right hand side describes ideal quasiparticles and the second one stands for the scattering contribution of quasiparticles. Here, the self energy functions Σ have to be expressed as a functional of f^Q .

In order to get an equation for f^Q , one can use Eq. (3.2) together with (3.12). As the quantity X (3.11) is of first order in the derivatives (it consists of a collision term), it can be neglected on the left hand side of (3.2). The δ function in (3.12) leads to the quasiparticle energies (2.10). Then $\text{Re}\Sigma$ does not depend on the variables \mathbf{R} or \mathbf{p} explicitly only but also implicitly via $\omega = E(\mathbf{p}, \mathbf{R}, T)$ [7]. It follows

$$\begin{aligned} 2\pi\delta(\omega - E) &\left[\frac{\partial}{\partial T} + \frac{\partial E}{\partial \mathbf{p}} \frac{\partial}{\partial \mathbf{R}} - \frac{\partial E}{\partial \mathbf{R}} \frac{\partial}{\partial \mathbf{p}}\right] f^Q(\mathbf{p}, \mathbf{R}, T) \\ &= (\pm i)\Sigma^<(\mathbf{p}, \omega, \mathbf{R}, T) ig^>(\mathbf{p}, \omega, \mathbf{R}, T) \\ &- i\Sigma^>(\mathbf{p}, \omega, \mathbf{R}, T) (\pm i)g^<(\mathbf{p}, \omega, \mathbf{R}, T). \end{aligned} \quad (3.15)$$

Integration with respect to ω gives

$$\begin{aligned} & \left[\frac{\partial}{\partial T} + \frac{\partial E}{\partial \mathbf{p}} \frac{\partial}{\partial \mathbf{R}} - \frac{\partial E}{\partial \mathbf{R}} \frac{\partial}{\partial \mathbf{p}} \right] f^Q(\mathbf{p}, \mathbf{R}, T) \\ &= \int \frac{d\omega}{2\pi} [(\pm i)\Sigma^<(\mathbf{p}, \omega, \mathbf{R}, T)ig^>(\mathbf{p}, \omega, \mathbf{R}, T) \\ & \quad - i\Sigma^>(\mathbf{p}, \omega, \mathbf{R}, T)(\pm i)g^<(\mathbf{p}, \omega, \mathbf{R}, T)] \\ &= \int \frac{d\omega}{2\pi} [(\pm i)\Sigma^<(\mathbf{p}, \omega, \mathbf{R}, T)ig^>(\mathbf{p}, \omega, \mathbf{R}, T) \\ & \quad - i\Sigma^>(\mathbf{p}, \omega, \mathbf{R}, T)(\pm i)Q^<(\mathbf{p}, \omega, \mathbf{R}, T)]. \quad (3.16) \end{aligned}$$

Finally, if we take into account the reconstruction formulas (3.9) and (3.10) in Eq. (3.16), a closed kinetic equation for the quasiparticle distribution function f^Q can be obtained. Then, the full Wigner distribution function could be calculated from (3.12) using (3.16). But, using a cluster expansion in terms of undamped quasiparticles, we get the well-known secular divergent terms. This is simply to be seen if one uses the reconstruction formulas (3.12) for the correlation function g^{\cong} in the skeleton diagrams of the self energy (e.g., the simplest form $\Sigma^{\cong} = V_{12}g_1^{\cong}g_2^{\cong}V_{12}g_2^{\cong}$). Then there appear divergent terms corresponding to successive binary collisions.

But, there is the other possibility to find a kinetic equation from which the Wigner distribution can be determined in a direct way. To do that, we have to express the correlation function $g^>$ and $g^<$ in terms of f instead of f^Q . This can be done in the way that the quasiparticle distribution function in (3.12) is substituted by the full Wigner function with the help of Eq. (3.13). In this way, the divergencies are compensated.

$$\begin{aligned} \pm ig^<(\omega, T) &= 2\pi\delta(\omega - E)f(T) - P' \frac{1}{\omega - E} \frac{\partial}{\partial T} f(T) \\ & \quad - P' \frac{1}{(\omega - E)} (\pm i)\Sigma^<(\omega, T) \\ & \quad + 2\pi\delta(\omega - E) \int \frac{d\bar{\omega}}{2\pi} P' \frac{1}{(\bar{\omega} - E)} (\pm i) \\ & \quad \times \Sigma^<(\bar{\omega}, T). \quad (3.17) \end{aligned}$$

Here, we have to keep in mind that the difference of the time derivatives of f^Q and f is of higher order.

Now we consider the Kadanoff Baym equations in first-order gradient expansion in the form (3.2). Integration with respect to ω leads to

$$\begin{aligned} & \frac{\partial f}{\partial T} - \frac{\partial}{\partial T} \int \frac{d\omega}{2\pi} \left[(\pm i)g^< \frac{\partial \text{Re}\Sigma^R}{\partial \omega} - (\pm i)\Sigma^< \frac{\partial \text{Reg}^R}{\partial \omega} \right] \\ &= \pm \int \frac{d\omega}{2\pi} (g^<\Sigma^> - g^>\Sigma^<). \quad (3.18) \end{aligned}$$

Using the dispersion relations for Reg and $\text{Re}\Sigma$, this equation can be written as

$$\frac{\partial f}{\partial T} = I^0(p, T) + I^1(p, T) \quad (3.19)$$

with

$$\begin{aligned} I^0(p_1 T) &= \int \frac{d\omega}{2\pi} [(\pm i)\Sigma^<(\omega T)ig^>(\omega T) \\ & \quad - i\Sigma^>(\omega T)(\pm i)g^<(\omega T)] \quad (3.20) \end{aligned}$$

and

$$\begin{aligned} I^1(p_1 T) &= \frac{\partial}{\partial T} \int \frac{d\omega_1 d\omega_2}{(2\pi)^2} P' \frac{1}{\omega_1 - \omega_2} \\ & \quad \times [i\Sigma^>(\omega_1 T)(\pm i)g^<(\omega_2 T) \\ & \quad - (\pm i)\Sigma^<(\omega_1 T)ig^>(\omega_2 T)]. \quad (3.21) \end{aligned}$$

Equation (3.21) represents a kinetic equation for the full Wigner distribution function. On the right hand side, the collision integrals are given in terms of the one-particle correlation functions g^{\cong} and the self-energy functions Σ^{\cong} . The latter are, again, functionals of the g^{\cong} . Therefore, using the reconstruction formula (3.17) in I^0 and I^1 , one gets a closed kinetic equation for the Wigner distribution function f . In comparison to the kinetic equation (3.16) for the quasiparticle distribution function, we have in (3.18) the additional collision integral I^1 which reflects nonideality corrections.

Explicit expressions for the quantities to be conserved at the level of approximation applied for the derivation of the kinetic equation follow from Eqs. (3.12)–(3.17). Using relation (3.14), we get for the density

$$\begin{aligned} n(\mathbf{R}, T) &= \int \frac{d^3 p}{(2\pi)^3} f(\mathbf{p}, \mathbf{R}, T) = \int \frac{d^3 p}{(2\pi)^3} f^Q(\mathbf{p}, \mathbf{R}, T) \\ & \quad + \int \frac{d\omega d^3 p}{(2\pi)^4} P' \frac{1}{\omega - E} \{i\Sigma^>(\mathbf{p}, \omega, \mathbf{R}, T)f^Q(\mathbf{p}, \mathbf{R}, T) \\ & \quad - (\pm i)\Sigma^<(\mathbf{p}, \omega, \mathbf{R}, T)[1 \pm f^Q(\mathbf{p}, \mathbf{R}, T)]\}, \quad (3.22) \end{aligned}$$

and with (3.17), we get for the energy

$$\begin{aligned} \langle \mathcal{E} \rangle &= \int \frac{d\omega}{(2\pi)} \frac{d^3 p}{(2\pi)^3} \frac{\omega + \frac{p^2}{2m}}{2} (\pm i)g^<(\mathbf{p}, \omega, \mathbf{R}, T) \\ &= \int \frac{d^3 p}{(2\pi)^3} \frac{p^2}{2m} f(\mathbf{p}, \mathbf{R}, T) \\ & \quad + \frac{1}{2} \int \frac{d^3 p}{(2\pi)^3} \text{Re}\Sigma(\mathbf{p}, E, \mathbf{R}, T)f(\mathbf{p}, \mathbf{R}, T) \\ & \quad + \frac{1}{2} \int \frac{d\omega d^3 p}{(2\pi)^4} P' \frac{1}{(\omega - E)} (\pm i)\Sigma^<(\omega, T), \quad (3.23) \end{aligned}$$

where we used the relation $(\omega - E)(\partial/\partial\omega)P[1/(\omega - E)] = -P[1/(\omega - E)]$.

With the dispersion relation for $\text{Re}\Sigma$, there follows

$$\begin{aligned} \langle \mathcal{E} \rangle &= \int \frac{d^3 p}{(2\pi)^3} \frac{p^2}{2m} f(\mathbf{p}, \mathbf{R}, T) \\ & \quad + \int \frac{d^3 p}{(2\pi)^3} \frac{\Sigma^{HF}(\mathbf{p}, \mathbf{R}, T)}{2} f(\mathbf{p}, \mathbf{R}, T) \\ & \quad + \frac{1}{2} \int \frac{d\omega d^3 p}{(2\pi)^4} P' \frac{1}{E - \omega} \{i\Sigma^>(\mathbf{p}, \omega, \mathbf{R}, T)f(\mathbf{p}, \mathbf{R}, T) \\ & \quad - (\pm i)\Sigma^<(\mathbf{p}, \omega, \mathbf{R}, T)[1 \pm f(\mathbf{p}, \mathbf{R}, T)]\}. \quad (3.24) \end{aligned}$$

Of course, in order to derive explicit kinetic equations from (3.19), one has to choose appropriate approximations for the self-energy functions Σ^{\approx} . Using standard approximations in I^0 , we can derive the well-known kinetic equations of Landau [25] (first Born approximation), Boltzmann (binary collision approximation), and Lenard and Balescu [26,27] (random phase approximation for the screening). But the kinetic equation (3.19) is very general and can serve as a starting point to go beyond these well-known approximations. First, cluster expansions for Σ^{\approx} can be used in I^0 to derive kinetic equations including three-particle scattering and reaction processes [20]. But, in spite of these higher-order approximations, the resulting kinetic equations remain valid for ideal systems only, i.e., ideal conservation laws only are fulfilled by these equations. To obtain kinetic equations for nonideal quantum systems, the retardation term I^1 , along with the corresponding gradient expansion correction of Σ in I^0 , must be taken into account. This will be shown in the next section considering the frequently used binary collision approximation.

IV. BINARY COLLISION APPROXIMATION

The binary collision approximation is a standard approximation of many-particle theory. The self-energy in this approximation reads with help of the T matrix

$$\Sigma^{\approx}(\mathbf{r}_1 t, \mathbf{r}'_1 t') = \int d^3 r_1 d^3 r_2 \langle \mathbf{r}_1 \mathbf{r}_2 | T^{\approx}(t, t') | \bar{\mathbf{r}}_2 \mathbf{r}'_1 \rangle \times (\pm i) g^{\approx}(\bar{\mathbf{r}}_2 t', \mathbf{r}_2 t). \quad (4.1)$$

The connection of the T matrix with the two-particle Green's function in binary collision approximation is given in the Appendix together with various useful relations. In this section, we will assume that no bound states are possible in the system. The Fourier transform of (4.1) is (for spatially homogeneous systems)

$$\Sigma^{\approx}(\mathbf{p}_1 \omega, T) = \int \frac{d^3 p_2}{(2\pi)^3} \frac{d\bar{\omega}}{2\pi} \langle \mathbf{p}_1 \mathbf{p}_2 | T^{\approx}(\omega + \bar{\omega}, T) | \mathbf{p}_2 \mathbf{p}_1 \rangle (\pm i) g^{\approx}(\mathbf{p}_2 \bar{\omega}, T). \quad (4.2)$$

Again we will use an operator notation in which (4.2) reads

$$\Sigma^{\approx}(\omega, T) = \pm i \text{Tr}_2 \int \frac{d\bar{\omega}}{2\pi} T^{\approx}_{12}(\omega + \bar{\omega}, T) g^{\approx}(\bar{\omega}, T), \quad (4.3)$$

with Tr_2 denoting the trace with respect to the second particle.

Now the collision integrals I^0 and I^1 can be expressed by the T matrices

$$I^0 = \text{Tr}_2 \int \frac{d\omega}{2\pi} \{ iT^{<}(\omega, T) i\mathcal{G}^>_{12}(\omega, T) - iT^{>}(\omega, T) i\mathcal{G}^<_{12}(\omega, T) \}, \quad (4.4)$$

and

$$I^1 = \frac{\partial}{\partial T} \text{Tr}_2 \int \frac{d\omega_1 d\omega_2}{(2\pi)^2} \text{P}' \frac{1}{\omega_1 - \omega_2} \{ iT^{>}(\omega_1, T) i\mathcal{G}^<_{12}(\omega_2, T) \}$$

$$- iT^{<}(\omega_1, T) i\mathcal{G}^>_{12}(\omega_2, T) \}, \quad (4.5)$$

where the two-particle quantity $\mathcal{G}^{\approx}_{12}$ was introduced by

$$\mathcal{G}^{\approx}_{12}(\omega, T) = i \int \frac{d\bar{\omega}}{2\pi} g^{\approx}(\omega - \bar{\omega}, T) g^{\approx}_2(\bar{\omega}, T). \quad (4.6)$$

The T^{\approx} matrices are given by the generalized optical theorem

$$T^{\approx}(t, t') = \int_{-\infty}^{\infty} d\bar{t} d\bar{t}' T^R(t, \bar{t}) \mathcal{G}^{\approx}_{12}(\bar{t}, \bar{t}') T^A(\bar{t}', t'). \quad (4.7)$$

Thus they can be expressed in terms of the correlation functions \mathcal{G}^{\approx} and the retarded and advanced T matrices $T^{R/A}$ which describe the in-medium scattering. The latter have to be determined from a generalized Lippmann-Schwinger equation (A5).

Our aim is to find the complete first-order corrections to the local approximations of the collision integral (usual Boltzmann collision integral). In the collision integral I^0 , we have to consider first-order contributions in \mathcal{G}^{\approx} [using the reconstruction formula (3.17) in (4.6)] and in the optical theorem as well; see Appendix.

If a nondegenerate system is considered, contributions of the kind $(\partial/\partial T)T^{R/A}$ can be neglected. Then it follows from (A9)

$$\begin{aligned} iT^{\approx}(\omega, T) &= T^R(\omega) 2\pi \delta(\omega - \bar{E}_{12}) \bar{F}^{\approx}_{12} T^A(\omega) \\ &\quad - T^R(\omega) \text{P}' \frac{1}{\omega - \bar{E}_{12}} \frac{\partial \bar{F}^{\approx}}{\partial T} T^A(\omega) + i\pi \delta(\omega - \bar{E}_{12}) \\ &\quad \times \left\{ \frac{\partial T^R(\omega)}{\partial \omega} \frac{\partial \bar{F}^{\approx}}{\partial T} T^A(\omega) \right. \\ &\quad \left. - T^R(\omega) \frac{\partial \bar{F}^{\approx}}{\partial T} \frac{\partial T^A(\omega)}{\partial \omega} \right\}. \end{aligned} \quad (4.8)$$

Here we have $F^{\approx}_{12} = (1 \pm f_1)(1 \pm f_2) \approx 1$, $F^<_{12} = f_1 f_2$ and $E_{12} = E_1 + E_2$. The first right hand side term of (4.8) is the well known local version of the optical theorem. The next contribution is the first-order gradient correction.

The collision term I^0 is easily obtained now as

$$\begin{aligned} I^0 &= I^B - \text{Tr}_2 \left\{ T^R(E_{12}) \text{P}' \frac{1}{E_{12} - \bar{E}_{12}} T^A(E_{12}) \frac{d\bar{F}^<}{dT} \right. \\ &\quad \left. - T^R(\bar{E}_{12}) \text{P}' \frac{1}{E_{12} - \bar{E}_{12}} T^A(\bar{E}_{12}) \frac{dF^<}{dT} - i\pi \delta(E_{12} - \bar{E}_{12}) \right. \\ &\quad \left. \times \left(\frac{\partial T^R}{\partial E_{12}} T^A - T^R \frac{\partial T^A}{\partial E_{12}} \right) \frac{d\bar{F}^<}{dT} \right\} \end{aligned} \quad (4.9)$$

with I^B being the quantum Boltzmann collision integral given by the expression local in time.

The collision integral I^1 contains already a time derivative. That is why the quantities in it can be taken in the lowest order, and we get

$$I^1 = \frac{\partial}{\partial T} \text{Tr}_2 T^R(\bar{E}_{12}) \mathbf{P}' \frac{1}{E_{12} - \bar{E}_{12}} T^A(\bar{E}_{12}) [\bar{F}^> F^< - \bar{F}^< F^>]. \quad (4.10)$$

The additional terms to I^B are essentially determined by the ‘‘off-shell’’ T matrix. For this quantity, we may derive a useful relation from the so-called differentiated optical theorem [see Appendix, especially (A14)]. Using the dispersion relation for $\text{Re}T(E)$, the following equation may be derived:

$$\begin{aligned} & T^R(\bar{E}_{12}) \mathbf{P}' \frac{1}{E_{12} - \bar{E}_{12}} \bar{N}_{12} T^A(\bar{E}_{12}) \\ &= T^R(E_{12}) \mathbf{P}' \frac{1}{E_{12} - \bar{E}_{12}} \bar{N}_{12} T^A(E_{12}) \\ &\quad - 2\pi \delta(E_{12} - \bar{E}_{12}) \bar{N}_{12} \text{Im}(T^R T^A) \end{aligned} \quad (4.11)$$

with $2 \text{Im}(T^R T^A) = i(T^{R'} T^A - T^R T^{A'})$ and $N_{12} = 1 \pm f_1 \pm f_2$ which is now $N_{12} \approx 1$.

Now we denote all corrections beyond the Boltzmann collision integral by I^R , and write the kinetic equation in the shape

$$\frac{df}{dT} = I^B(p_1) + I^R(p_1). \quad (4.12)$$

The Boltzmann collision integral reads

$$\begin{aligned} I^B &= \int \frac{d^3 p_2 d^3 \bar{p}_1 d^3 \bar{p}_2}{(2\pi)^9} |\langle \mathbf{p}_1 \mathbf{p}_2 | T(E_{12} + i\varepsilon) | \bar{\mathbf{p}}_2 \bar{\mathbf{p}}_1 \rangle|^2 \\ &\quad \times 2\pi \delta(E_{12} - \bar{E}_{12}) (\bar{f}_1 \bar{f}_2 - f_1 f_2). \end{aligned} \quad (4.13)$$

The first-order retardation terms collected in $I^R(p_1)$ are given by

$$\begin{aligned} I^R(p_1) &= \frac{d}{dT} \int \frac{d^3 p_2 d^3 \bar{p}_1 d^3 \bar{p}_2}{(2\pi)^9} \left\{ [|\langle \mathbf{p}_1 \mathbf{p}_2 | T(\bar{E}_{12}) | \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 \rangle|^2 \right. \\ &\quad + |\langle \mathbf{p}_1 \mathbf{p}_2 | T(E_{12}) | \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 \rangle|^2] \mathbf{P}' \frac{1}{E_{12} - \bar{E}_{12}} \\ &\quad \left. - 2\pi \delta(E_{12} - \bar{E}_{12}) \text{Im}(T^R T^A) \right\} \left(\frac{dF^<}{dT} - \frac{d\bar{F}^<}{dT} \right). \end{aligned} \quad (4.14)$$

Now we show that there is an interesting relation between I^B and I^R . For this reason we define

$$\begin{aligned} I^B(\varepsilon) &= \int \frac{d^3 p_2 d^3 \bar{p}_1 d^3 \bar{p}_2}{(2\pi)^9} \int \frac{d\omega}{2\pi} \frac{2\varepsilon}{(E - \omega)^2 + \varepsilon^2} \\ &\quad \times \frac{2\varepsilon}{(\bar{E} - \omega)^2 + \varepsilon^2} |\langle \mathbf{p}_1 \mathbf{p}_2 | T(\omega + i\varepsilon) | \bar{\mathbf{p}}_2 \bar{\mathbf{p}}_1 \rangle|^2 \\ &\quad \times (\bar{F}_{12}^< - F_{12}^<). \end{aligned} \quad (4.15)$$

Then we see easily

$$I^B(p_1) = \lim_{\varepsilon \rightarrow 0} I^B(p_1, \varepsilon),$$

and with

$$\lim_{\varepsilon \rightarrow 0} \frac{d}{d\varepsilon} \frac{\varepsilon}{x^2 + \varepsilon^2} = \mathbf{P}' \frac{1}{x},$$

we get

$$I^R = \frac{1}{2} \frac{d}{dT} \frac{d}{d\varepsilon} I^B(\varepsilon) \Big|_{\varepsilon \rightarrow 0}. \quad (4.16)$$

The kinetic equation in first-order retardation has the compact shape

$$\frac{d}{dT} f(p_1 T) = \left(1 + \frac{1}{2} \frac{d}{dT} \frac{d}{d\varepsilon} \right) I^B(p_1, \varepsilon, T) \Big|_{\varepsilon \rightarrow 0}. \quad (4.17)$$

The investigation of conservation laws, inherent in the kinetic equation, is a useful means in order to control the level of approximation [28–30]. We expect conservation laws for the total number density $n(\mathbf{R}, T)$ (3.22), for the momentum and for the total energy $\langle \mathcal{E} \rangle$ (3.23). It is well known that the Markovian kinetic equation conserves only the average kinetic energy, what is insufficient for strongly correlated systems. This motivates the inclusion of retardation effects.

Bärwinkel [2] determined retardation corrections to the Boltzmann collision integral, which consist of two contributions [formula (13) of [2]]. One of those contributions [the term determined by $E(\mathbf{p}_1 \mathbf{p}_2 \mathbf{p}'_1 \mathbf{p}'_2)$] coincides with ours (4.14). The second term denoted by $O(\mathbf{p}_1 \mathbf{p}_2 \mathbf{p}'_1 \mathbf{p}'_2)$ should not appear. This is due to the fact that Bärwinkel used the Kadanoff-Baym ansatz instead of (3.12), thus the retardation was considered only partly. Consequently, not the total energy is conserved but only part of it.

Let us now consider this problem from the point of view of the generalized kinetic equation investigated in the previous sections. Now we consider explicit expressions for the density. In binary collision approximation, we get using equations of the previous sections,

$$\begin{aligned} n(\mathbf{R}, T) &= \int \frac{d^3 p}{(2\pi)^3} f^Q(\mathbf{p}, \mathbf{R}, T) + \int \frac{d^3 p_1 d^3 p_2 d^3 \bar{p}_1 d^3 \bar{p}_2}{(2\pi)^{12}} \\ &\quad \times \mathbf{P}' \frac{1}{E_{12} - \bar{E}_{12}} \langle \mathbf{p}_1 \mathbf{p}_2 | T(\bar{E}_{12} + i\varepsilon) | \bar{\mathbf{p}}_2 \bar{\mathbf{p}}_1 \rangle \\ &\quad \times \langle \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 | T(\bar{E}_{12} - i\varepsilon) | \mathbf{p}_2 \mathbf{p}_1 \rangle [\bar{F}_{12}^< - F_{12}^<]. \end{aligned} \quad (4.18)$$

Using (4.11) and the symmetry with respect to the interchange $\mathbf{p}_1, \mathbf{p}_2 \leftrightarrow \bar{\mathbf{p}}_1, \bar{\mathbf{p}}_2$, we have only on-shell quantities

$$\begin{aligned} n(\mathbf{R}, T) &= \int \frac{d^3 p}{(2\pi)^3} f^Q(\mathbf{p}, \mathbf{R}, T) \\ &\quad + \int \frac{d^3 p_1 d^3 p_2 d^3 \bar{p}_1 d^3 \bar{p}_2}{(2\pi)^{12}} i\pi \delta(E_{12} - \bar{E}_{12}) \\ &\quad \times \left\{ \frac{\partial}{\partial E} [\langle \mathbf{p}_1 \mathbf{p}_2 | T_{12}^R(E) | \bar{\mathbf{p}}_2 \bar{\mathbf{p}}_1 \rangle \langle \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 | T_{12}^A(E) | \mathbf{p}_2 \mathbf{p}_1 \rangle \right. \\ &\quad \left. - \langle \mathbf{p}_1 \mathbf{p}_2 | T_{12}^A(E) | \bar{\mathbf{p}}_2 \bar{\mathbf{p}}_1 \rangle \frac{\partial}{\partial E} \right. \\ &\quad \left. \times [\langle \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 | T_{12}^A(E) | \mathbf{p}_2 \mathbf{p}_1 \rangle] \right\} F_{12}^<. \end{aligned} \quad (4.19)$$

The first term on the right hand side may be considered as the density of quasiparticles whereas the second is the correlation part

$$n(\mathbf{R}, T) \equiv n^Q(\mathbf{R}, T) + n^{\text{corr}}(\mathbf{R}, T). \quad (4.20)$$

Expression (4.19) represents a cluster expansion for the density in binary collision approximation.

Let us now show that the density $n(\mathbf{R}, T)$ given by (4.18) or (4.19), respectively, is conserved by the kinetic equation (3.19). It follows from (3.19) with (3.20) and (3.21) after integration with respect to p_1 ,

$$\begin{aligned} \frac{d}{dT} \int \frac{d^3 p_1}{(2\pi)^3} f(\mathbf{p}_1, \mathbf{R}, T) &= \frac{dn}{dT} = \frac{d}{dT} [n^Q(\mathbf{R}, T) + n^{\text{corr}}(\mathbf{R}, T)] \\ &= \int \frac{d^3 p_1}{(2\pi)^3} I^0 + \int \frac{d^3 p_1}{(2\pi)^3} I^1. \end{aligned} \quad (4.21)$$

Extracting first-order retardation terms from I^0 , we have

$$\frac{d}{dT} n(\mathbf{R}, T) = \int \frac{d^3 p_1}{(2\pi)^3} I^B + \int \frac{d^3 p_1}{(2\pi)^3} I^R. \quad (4.22)$$

It is well known since Boltzmann that the first integral vanishes. From the explicit expression for I^R given by (4.14), it follows using relation (4.11) for the connection between off-shell and on-shell quantities that the second integral vanishes, too. Thus we have

$$\frac{d}{dT} (n^Q + n^{\text{corr}}) = \frac{dn}{dT} = 0. \quad (4.23)$$

This is the conservation law for the thermodynamic quantity ‘‘density of the nonideal system.’’ The corresponding consideration in a paper recently published [31] differs from ours. In [31], the retardation correction was reduced only to I^1 , and the quantity $\int f(p) [d^3 p / (2\pi)^3]$ was identified with the quasiparticle density what is incomplete.

Let us now consider the energy conservation. This problem is more complex. We start with the construction of an equation for the average energy (3.23) or (3.24). We consider again Eq. (4.12), multiply it with the kinetic energy, and take the trace over 1. Symmetrization with respect to 1 and 2, and $\bar{1}$ and $\bar{2}$, leads to the simple result for the derivation of the kinetic energy T ,

$$\frac{\partial}{\partial T} \langle T \rangle = -\frac{1}{4} \frac{\partial}{\partial T} \text{Tr}_{12} \left[|T^R(\bar{E})|^2 \mathbf{P} \frac{1}{E - \bar{E}} (2F_{12}^< - 2\bar{F}_{12}^<) \right]. \quad (4.24)$$

The mean potential energy is given by the second and third terms of the right hand side of (3.24). If the self-energy is expressed in terms of T matrices, we have for the potential energy (cf. [32])

$$\langle V \rangle = \langle V \rangle^{HF} + \frac{1}{2} \text{Tr}_{12} \left[|T^R(\bar{E})|^2 \mathbf{P} \frac{1}{E - \bar{E}} (F_{12}^< - \bar{F}_{12}^<) \right] \quad (4.25)$$

and we get

$$\frac{\partial}{\partial T} \langle T \rangle = -\frac{\partial}{\partial T} \langle V \rangle, \quad (4.26)$$

i.e.,

$$\frac{\partial}{\partial T} \langle H \rangle = \frac{\partial}{\partial T} \langle T + V \rangle = 0, \quad (4.27)$$

which means conservation of the total energy.

V. SYSTEM WITH BOUND STATES

The problem of bound states in kinetic theory is the subject of many papers, e.g., [29,33–36]. On the time scale of two-particle scattering processes, bound states are long-living entities. Thus, e.g., one has to account for the bound state part of the two-particle T matrix in the kinetic equation.

We recall the expression for Σ in the binary collision approximation,

$$\begin{aligned} \Sigma^{\cong}(\mathbf{p}_1, \omega, T) &= \int \frac{d^3 p_2}{(2\pi)^3} \frac{d\bar{\omega}}{2\pi} \langle \mathbf{p}_1 \mathbf{p}_2 | T^{\cong}(\omega + \bar{\omega}, T) | \mathbf{p}_2 \mathbf{p}_1 \rangle \\ &\times (\pm i) g^{\cong}(\mathbf{p}_2, \bar{\omega}, T). \end{aligned} \quad (5.1)$$

Bound state contributions are possible if the energy argument of the T matrix is taken off the energy shell of two free particles. The T matrix and the two-particle correlation function are connected by

$$T^{\cong}(\omega, T) = iV g_{12}^{\cong}(\omega, T) V. \quad (5.2)$$

The two-particle correlation function can have a scattering part and a bound state part: $g_{12} = g_{12}^{\text{scatt}} + g_{12}^{\text{bound}}$. It is possible to use a bilinear expansion [37,38] for the two-particle Green’s function. The time dependence is considered in the local approximation,

$$g_{12}^{\cong}(\omega, T) = \sum_K |\Psi^K\rangle \langle \Psi^K | N_K^{\cong} 2\pi \delta(\omega - E_K) \quad (5.3)$$

with wave function $|\Psi^K\rangle$ and energy eigenvalues E_K following from the effective Schrödinger equation with the time argument being suppressed,

$$\begin{aligned} [E_K - \epsilon(p_1) - \epsilon(p_2)] \langle \mathbf{p}_1 \mathbf{p}_2 | \Psi^K \rangle - [1 - f(p_1) - f(p_2)] \\ \times \int \frac{d\bar{p}_1 d\bar{p}_2}{(2\pi)^6} \langle \mathbf{p}_1 \mathbf{p}_2 | V | \bar{\mathbf{p}}_2 \bar{\mathbf{p}}_1 \rangle \langle \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 | \Psi^K \rangle = 0. \end{aligned} \quad (5.4)$$

This equation is not Hermitian but one can construct a biorthonormal system $|\tilde{\Psi}^K\rangle$ from

$$[E_K - \epsilon(p_1) - \epsilon(p_2)] \langle \mathbf{p}_1 \mathbf{p}_2 | \tilde{\Psi}^K \rangle - \int \frac{d\bar{p}_1 d\bar{p}_2}{(2\pi)^6} \langle \mathbf{p}_1 \mathbf{p}_2 | V | \bar{\mathbf{p}}_2 \bar{\mathbf{p}}_1 \rangle \times [1 - f(\bar{p}_1) - f(\bar{p}_2)] \langle \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 | \tilde{\Psi}^K \rangle = 0 \quad (5.5)$$

with

$$\langle \psi^{K'} | \tilde{\Psi}^K \rangle = \delta_{KK'} \sum_K |\tilde{\Psi}^K\rangle \langle \psi^K| = 1. \quad (5.6)$$

Further it holds that

$$\langle \mathbf{p}_1 \mathbf{p}_2 | \tilde{\Psi}^K \rangle = \frac{\mathcal{F}_K}{1 - f(p_1) - f(p_2)} \langle \mathbf{p}_1 \mathbf{p}_2 | \Psi^K \rangle \quad (5.7)$$

with

$$\mathcal{F}_K^{-1} = \int \frac{d\bar{p}_1 d\bar{p}_2}{(2\pi)^6} \langle \Psi^K | \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 \rangle \frac{1}{1 - f(\bar{p}_1) - f(\bar{p}_2)} \langle \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 | \tilde{\Psi}^K \rangle. \quad (5.8)$$

$N_K^<$ and $N_K^>$ are connected by $N_K^> - N_K^< = \mathcal{F}_K$. Setting $N_K^< = N_K \mathcal{F}_K$, the quantity N_K is the distribution function of the two-particle bound and scattering states, respectively. In equilibrium, we have

$$N_K = \frac{1}{e^{\beta(E_K - \mu_a - \mu_b)} - 1}. \quad (5.9)$$

In the kinetic equation, the bound state contribution appears in the collision integral I^1 , which can be written in the form

$$I^1 = \frac{\partial}{\partial T} \int \frac{dp_2}{(2\pi)^3} \times \left\{ \left(\frac{\partial}{\partial \omega} \text{Re} \langle \mathbf{p}_1 \mathbf{p}_2 | T^R(\omega) | \mathbf{p}_2 \mathbf{p}_1 \rangle \right) \Big|_{\omega = \epsilon(p_1) + \epsilon(p_2)} f_1 f_2 - \int \frac{d\omega}{2\pi} \langle \mathbf{p}_1 \mathbf{p}_2 | T^<[\omega + \epsilon(p_2)] | \mathbf{p}_2 \mathbf{p}_1 \rangle P' \frac{1}{\omega - \epsilon(p_1)} \times (1 - f_1 - f_2) \right\}. \quad (5.10)$$

In the second contribution at the right hand side of Eq. (5.10), the T matrix has to be taken off-shell. Inserting the bilinear expansion for the T matrix, we can separate the bound state part and get

$$I^1 = I_{scatt}^1 + \frac{\partial}{\partial T} \int \frac{dp_2}{(2\pi)^3} \sum_{jP} \langle \mathbf{p}_1 \mathbf{p}_2 | \Psi^{jP} \rangle \langle \tilde{\Psi}^{jP} | \mathbf{p}_2 \mathbf{p}_1 \rangle N_j(P) \quad (5.11)$$

with j, P being the set of quantum numbers of bound states. The term I_{scatt}^1 is the same as in Sec. IV.

We introduce the distribution function of the unbound particles by

$$f^F(p_1) = f(p_1) - \int \frac{dp_2}{(2\pi)^3} \sum_{jP} \langle \mathbf{p}_1 \mathbf{p}_2 | \Psi^{jP} \rangle \times \langle \tilde{\Psi}^{jP} | \mathbf{p}_2 \mathbf{p}_1 \rangle N_j(P). \quad (5.12)$$

For nonequilibrium systems, the distribution functions f^F and N_j are independent. The temporal change of the distribution function of bound states, which is due to three-particle processes, has to be calculated from a separate kinetic equation. Such an equation can be derived from the second equation of the Martin-Schwinger hierarchy and reads [36,39,40]

$$\frac{d}{dT} N_j(P, T) = I_j^{scatt} + I_j^{rearr} + I_j^{react}. \quad (5.13)$$

Thus, in order to fulfill, e.g., the conservation of the total density, for the self-energy in the zeroth-order term I^0 , a cluster expansion has to be used which includes also three-particle processes. The kinetic equation for f^F reads then

$$\frac{d}{dT} f^F(p_1, T) = I_B + I_3^0 + I^R \quad (5.14)$$

with I_3^0 being a Markovian three-particle collision integral including reactions. The temporal change of the corresponding density $n_F = \int [dp/(2\pi)^3] f^F(p)$ is given by a rate equation,

$$\frac{d}{dT} n_F = \int \frac{dp}{(2\pi)^3} I_3^0 = \sum_j \alpha_j n_F n_j - \beta_j n_F^3, \quad (5.15)$$

where for the density $n_j = \int [dP/(2\pi)^3] N_j(P)$ of bound states in level j holds

$$\frac{dn_j}{dT} = \beta_j n_F^3 - \alpha_j n_F n_j + \sum_j n_F n_j K_{jj}^- - n_F n_j K_{jj}^+. \quad (5.16)$$

It follows that the total density $n = n_F + \sum_j n_j$ is conserved. Note that n_F is given by [cf. (3.13)]

$$n_F(\mathbf{R}, T) = \int \frac{d^3p}{(2\pi)^3} f^Q(\mathbf{p}, \mathbf{R}, T) + \int \frac{d^3p_1 d^3p_2 d^3\bar{p}_1 d^3\bar{p}_2}{(2\pi)^{12}} i\pi \delta(E_{12} - \bar{E}_{12}) \times \left\{ \frac{\partial}{\partial E} [\langle \mathbf{p}_1 \mathbf{p}_2 | T_{12}^R(E) | \bar{\mathbf{p}}_2 \bar{\mathbf{p}}_1 \rangle] \times \langle \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 | T_{12}^A(E) | \mathbf{p}_2 \mathbf{p}_1 \rangle - \langle \mathbf{p}_1 \mathbf{p}_2 | T_{12}^A(E) | \bar{\mathbf{p}}_2 \bar{\mathbf{p}}_1 \rangle \frac{\partial}{\partial E} \times [\langle \bar{\mathbf{p}}_1 \bar{\mathbf{p}}_2 | T_{12}^A(E) | \mathbf{p}_2 \mathbf{p}_1 \rangle] \right\} f(p_1) f(p_2). \quad (5.17)$$

The first term on the right hand side may be considered as the density of quasiparticles whereas the second is the correlation part (scattering states). Thus we have

$$n(\mathbf{R}, T) = n^Q(\mathbf{R}, T) + n^{\text{corr}}(\mathbf{R}, T) + \sum_j n_j(\mathbf{R}, T). \quad (5.18)$$

Expression (5.18) represents a cluster expansion for the density in binary collision approximation including bound states.

This type of relation for thermodynamic quantities is well known in the case of equilibrium quantum statistics [15,37,41–44]. The connection between such thermodynamic expressions and Eq. (5.18) becomes obvious if we expand the first right hand side contribution of (5.18), i.e., the quasiparticle distribution $f(p^2/2m + \text{Re}\Sigma)$, with respect to $\text{Re}\Sigma^{\text{corr}}$. For $\text{Re}\Sigma^{\text{corr}}$ we use the approximation

$$\text{Re}\Sigma^{\text{corr}}(E) = \text{Tr}_2 \text{Re} T_{12}(E_1 + \bar{E}_2) f_2(\bar{E}_2). \quad (5.19)$$

Further, we introduce the partial wave expansion for the T matrix [32]. We get the generalized Beth-Uhlenbeck formula

$$\begin{aligned} n = & \int \frac{d^3p}{(2\pi)^3} f^{HF}(p) + \sum_{\mathcal{L}} \int \frac{d^3P}{(2\pi)^3} (2\mathcal{L}+1) \left[1 - \frac{(-1)^{\mathcal{L}}}{2s+1} \right] \\ & \times \left\{ \sum_n n^B \left(E_{n\mathcal{L}} + \frac{P^2}{2M} \right) + \frac{1}{\pi} \int_0^\infty dE n^B \left(\frac{p^2}{2m} + \frac{P^2}{2M} \right. \right. \\ & \left. \left. + \Delta_1 + \Delta_2 \right) \frac{d\delta_{\mathcal{L}}(E)}{dE} \right\}. \end{aligned} \quad (5.20)$$

Here $\Delta_{1,2}$ are single-particle energy shifts, n^B is the Bose function, and $\delta_{\mathcal{L}}$ is the scattering phase shift. In thermodynamic equilibrium, Eq. (5.20) determines completely the thermodynamic behavior of the system in the binary collision approximation.

VI. CONCLUSIONS

The present paper gives a derivation of a rather general kinetic equation in the framework of Green's functions techniques. Starting from the general Kadanoff-Baym equations, we did a first-order gradient expansion. In the ‘‘Poisson brackets’’ containing Reg^R , we made an approximation with respect to the damping Γ leading to a linearized spectral function having thus a quasiparticle and an off-pole part. Within this approximation, the correlation functions have a corresponding structure. In order to achieve kinetic equations, one has to solve the problem to express the correlation functions in terms of the distribution function. In detail, the differences between the Wigner and the quasiparticle distribution function were investigated.

Explicit collision integrals were formulated in the binary collision approximation. Besides the usual Boltzmann integral, there occur additional contributions which account for retardation effects. In contrast to the usual Markovian Boltzmann equation, the retardation effects lead to conservation laws of number density and energy; especially, the latter goes beyond the kinetic energy.

Bound states come into the play via the off-shell contributions of the retardation part of the collision integral. We want to mention that bound states do not occur in balances of the usual Boltzmann equation.

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APPENDIX: T -MATRIX APPROXIMATION

The two-particle Green's function g_{12} is given in ladder approximation by

$$\begin{aligned} i g_{12}^{\cong}(t, t') = & \mathcal{G}_{12}^{\cong}(t, t') + i \int_{-\infty}^{\infty} d\bar{t} \bar{\mathcal{G}}_{12}^R(t, \bar{t}) V g_{12}^{\cong}(\bar{t}, t') \\ & + i \int_{-\infty}^{\infty} d\bar{t} \bar{\mathcal{G}}_{12}^{\cong}(t, \bar{t}) V g_{12}^A(\bar{t}, t'), \end{aligned} \quad (A1)$$

where the abbreviation

$$\mathcal{G}_{12}^{\cong}(t, t') = i g_1^{\cong}(t, t') g_2^{\cong}(t, t') \quad (A2)$$

was introduced. Bound states are not considered here. The retarded (advanced) two-particle function is determined by

$$i g_{12}^{R/A}(t, t') = \mathcal{G}_{12}^{R/A}(t, t') + i \int_{-\infty}^{\infty} d\bar{t} \bar{\mathcal{G}}_{12}^{R/A}(t, \bar{t}) V g_{12}^{R/A}(\bar{t}, t'). \quad (A3)$$

If the T matrix is defined by

$$T^{\cong}(t, t') = i V g_{12}^{\cong}(t, t') V, \quad (A4)$$

$$T^{R/A}(t, t') = V \delta(t - t') + i V g_{12}^{R/A}(t, t') V,$$

its equation of motion follows from (A1),

$$\begin{aligned} T^{\cong}(t, t') = & \int_{-\infty}^{\infty} d\bar{t} \bar{V} \mathcal{G}_{12}^R(t, \bar{t}) T^{\cong}(\bar{t}, t') \\ & + \int_{-\infty}^{\infty} d\bar{t} \bar{V} \mathcal{G}_{12}^{\cong}(t, \bar{t}) T^A(\bar{t}, t'), \end{aligned} \quad (A5)$$

$$T^{R/A}(t, t') = V \delta(t - t') + \int_{-\infty}^{\infty} d\bar{t} \bar{V} \mathcal{G}_{12}^{R/A}(t, \bar{t}) T^{R/A}(\bar{t}, t'), \quad (A6)$$

which gives

$$T^{\cong}(t, t') = \int_{-\infty}^{\infty} d\bar{t} d\bar{t}' T^R(t, \bar{t}) \mathcal{G}_{12}^{\cong}(\bar{t}, \bar{t}') T^A(\bar{t}', t'). \quad (A7)$$

The latter equation can be considered as a generalized optical theorem.

It is of special interest to have the Fourier transform of (A7). The two-time quantities in (A7) do not only depend on the time differences. Thus, the Fourier transform $T^{\cong}(\omega, T)$ up to first order in the derivatives is given by

$$\begin{aligned} T^{\cong}(\omega, T) = & T^R(\omega, T) \mathcal{G}^{\cong}(\omega, T) T^A(\omega, T) + \frac{i}{2} \left[T^R \frac{\partial \mathcal{G}^{\cong}}{\partial \omega} \frac{\partial T^A}{\partial T} \right. \\ & - T^R \frac{\partial \mathcal{G}^{\cong}}{\partial T} \frac{\partial T^A}{\partial \omega} + \frac{\partial T^R}{\partial \omega} \frac{\partial \mathcal{G}^{\cong}}{\partial T} T^A - \frac{\partial T^R}{\partial T} \frac{\partial \mathcal{G}^{\cong}}{\partial \omega} T^A \\ & \left. + \frac{\partial T^R}{\partial \omega} \mathcal{G}^{\cong} \frac{\partial T^A}{\partial T} - \frac{\partial T^R}{\partial T} \mathcal{G}^{\cong} \frac{\partial T^A}{\partial \omega} \right]. \end{aligned} \quad (A8)$$

A further important relation is that for the derivative of the real part of $T(z)$. Starting from the expression for the imaginary part (optical theorem in lowest order with respect to time derivatives)

$$\begin{aligned} \text{Im}T(\omega+i\epsilon, T) &= -\frac{i}{2}[T(\omega+i\epsilon, T) - T(\omega-i\epsilon, T)] \\ &= -\frac{i}{2}[T^>(\omega, T) - T^<(\omega, T)] \\ &= T(\omega+i\epsilon, T)\text{Im}\mathcal{G}(\omega+i\epsilon, T) \\ &\quad \times T(\omega-i\epsilon, T) \end{aligned} \quad (\text{A9})$$

and keeping in mind

$$\frac{\partial}{\partial\omega}F(\omega+i\epsilon) = \frac{\partial}{\partial i\epsilon}F(\omega+i\epsilon), \quad (\text{A10})$$

which implies

$$\frac{\partial}{\partial\omega}\text{Re}F(\omega+i\epsilon) = \frac{\partial}{\partial\epsilon}\text{Im}F(\omega+i\epsilon), \quad (\text{A11})$$

$$\frac{\partial}{\partial\omega}\text{Im}F(\omega+i\epsilon) = -\frac{\partial}{\partial\epsilon}\text{Re}F(\omega+i\epsilon), \quad (\text{A12})$$

we get for the derivative of the real part

$$\frac{\partial}{\partial\omega}\text{Re}T(\omega+i\epsilon) = T\frac{\partial\text{Re}\mathcal{G}}{\partial\omega}T^* + i\frac{\partial T}{\partial\omega}\text{Im}\mathcal{G}T^* - iT\text{Im}\mathcal{G}\frac{\partial T^*}{\partial\omega}. \quad (\text{A13})$$

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